MA 7251: THEORY OF NUMBERS

(Prerequisite: Nil)

Total hours: 56

Module I: (14 hours)
Divisibility: properties, the Euclidean algorithm, primes, Fundamental Theorem of Arithmetic, the sum and the number of divisors of an integer, Mersenne and Fermat numbers, perfect numbers, linear Diophantine equations.

Module II: (14 hours)
Congruence: basic properties, divisibility criteria, linear congruences, Chinese Remainder Theorem, polynomial congruences of higher degree, Lagrange’s Theorem for a prime modulus, Euler phi-function, Theorems of Fermat and Euler-Fermat, Wilson’s Theorem, pseudo primes and Carmichael numbers.

Module III: (14 hours)
Primitive Roots, Index Arithmetic, Quadratic residues and reciprocity: quadratic residues, Euler’s criterion, Legendre and Jacobi symbols, the law of quadratic reciprocity.

Module IV: (14 hours)

References
MA7252: KNOT THEORY

(Prerequisite: Nil)

Total hours: 56

MODULE I: (14 hours)
Knots and Links: Knots and links through their diagrams, Basic Definitions- orientation, positive and negative crossing, writhe of a diagram, linking numbers, Isotopy of Knots-Polynomial links and Reidemeister Moves, Hopf not isotopic to Whitehead.

MODULE II: (14 hours)
Knot colouring, labelling arcs with integers, colouring mod n, splittable links, Borromean rings, chess board structure on a diagram, reduced connected diagram and quadrilateral decomposition, crossing equations, colouring through the determinant of a link , Invariant of Knots:- Numerical invariants: Crossing number-Unknotting number- Bridge number,

MODULE III: (14 hours)
Mirrors and Knot coding, mirrors, reversing orientation, chiral knots, Knots and Surfaces:- Seifert Surfaces- Polynomials-Alexander Polynomial-Jones Polynomial- bracket polynomial, state sums, skein relations, Conway knot and Kinoshita-Terasaka knot, Types of Knots - Torus knots- Satellite knots- Hyperbolic knots-

MODULE IV: (14 hours)
Topology of 3-manifolds Heegaard Splittings:- Lens Spaces, Seifert Manifolds, Morse Functions- Surgery on Links. Framing, linking number, homology of knot exterior. Surgery description of Lens Spaces and Seifert Manifolds

References

MA7253: MATHEMATICAL METHODS IN FLUID DYNAMICS

(Prerequisite: Knowledge of Calculus, Differential equations)

Total hours: 56

Module I: (15 hours)

Fundamental concepts about fluids, Some mathematical concepts and notation, Kinematic of fluids, The transport theorem, The continuity equation, The equations of motion, Stress Tensor, The Navier-Stokes equations, The Energy equations, Boundary conditions

Module II: (15 hours)

Equations of motion in different coordinate systems, Curvilinear coordinates, Cylindrical coordinates, Dimensional analysis, Bukingham’s \( \pi \) theorems, An example, Low Reynolds number flows, Steady and unsteady plane flows, Boussinesq approximation

Module III: (13 hours)

Rotation and vorticity, Kelvin’s circulation theorem, Potential flow, Stream line, Stream functions, Blasius’ theorem, Kutta-Joukowski theorem, Surface phenomena, Laplace’s formula, Capillarity

Module IV: (13 hours)

Boundary-Layers concept, Prandtl boundary layer equations, General properties and exact solutions of the boundary-layer equations for plane flows

References

MA724: ADVANCED COMPLEX ANALYSIS

(Pre-requisite: Knowledge of Complex Analysis)

Total Hours: 56

Module I: (13 hours)


Module II: (14 hours)


Module III: (12 hours)

Elliptic Functions, Simply periodic functions, Representation by exponents, Fourier Development, Functions of finite order, Doubly periodic functions, Period Module, Unimodular transformations, Cannonical Basis, General properties of elliptic functions.

Module IV: (17 hours)

Weierstrass Theory, Germs and Sheaves, Sections of Riemann Surfaces, Analytic continuation along arcs, Homotopic curves, The Monodromy theorem, Branch points, the Picard theorem, subharmonic functions, spaces $H^p$ and $N$, factorization theorems, shift operator, conjugate functions.

References

MA7255: STOCHASTIC MODELS IN OPERATIONS RESEARCH

(Pre-requisite: Probability Theory)

Total hours: 56

Module I: (14 hours)
Continuous time Markov Chains, Poisson Processes, Birth-Death processes, Queueing systems, general concepts, transient and steady state behavior, M/M/1 queue, system size and waiting time distributions, M/M/1/k model, M/M/∞, M/M/c models, Erlang loss models.

Module II: (14 hours)
Erlang queueing models, the system M/E_r/1, the system E_r/M/1, Bulk queues, network of Markovian queues, channels in series, Jackson networks. Non-Markovian queueing systems, the M/G/1 model and G/M/1 model, busy period analysis.

Module III: (14 hours)
Reliability theory: Introduction, structure functions, reliability of systems of independent components, the series system, the parallel system, the k-out-of-n system, bounds on the reliability function, systems with life as a function of component lives, expected system life time, systems with repair.

Module IV: (14 hours)
Inventory Theory: Introduction to basic inventory models, EOQ models, Stochastic inventory models, single period decision models, marginal analysis, the news vendor problem- discrete demand, continuous demand, the EOQ with uncertain demand, (s,S) models, safety stock, periodic review policy, the ABC inventory classification system.

REFERENCES:

MA7256: ADVANCED TOPICS IN GRAPH THEORY

(Pre-requisite: Nil)

Total hours: 42

Module I: (8 hours)

**Graphs:** review of basics in graphs - Trees- Blocks- Matrices-Operations on graphs, **Connectivity:** Vertex Connectivity and edge connectivity – n- connected graphs-Menger’s Theorem, **Traversability:** Euler graphs-Hamiltonian Graphs-Planar and Nonplanar graphs.

Module II: (14 hours)

**Metric in graph:** Centre, Median, eccentric vertex, Eccentric graph, boundary vertex, complete vertex, interior vertex, **Convexity:** Closure Invariants-gin(G) –gn(G)-Hull number- Geodetic Graphs- Distance Hereditary Graphs, **Symmetry:** Graphs and groups-- Symmetric Graphs - Distance Symmetry-Distance transitive graphs-distance regular graphs, **Distance Sequences:** Degree sequence, Eccentric Sequence - Distance Sequences - The Distance Distribution, Mean distance.

Module III: (10 hours)

**Matchings:** Maximum matching-Perfect matching-Matching in bipartite graphs, **Factorization:** Coverings and independence-1-factorization-2-factorization-Arboricity, **Domination:** Dominating set-Domination number-total dominating set –total domination number.

Module IV: (10 hours)

**Digraphs:** Digraphs and connectedness- Tournaments- directed trees-binary trees- weighted trees and prefix codes, **Networks:** Flows-cuts- The Max- Flow Min-Cut Theorem, **Graph Algorithms:** Polynomial Algorithms and NP completeness ,Complexity, Search algorithms, Shortest path algorithms.

**References**

MA7257: DIFFERENTIAL GEOMETRY

(Pre-requisite: Nil)

Total hours: 56

Module I (14 hours)


Module II (14 hours)


Module III (14 Hours)

Arc lengths and line integrals- Curvature of surfaces, Intrinsic equation of a curve, Linear element, Element of area, Intrinsic geometry.

Module IV (14 Hours)

Parameterized surfaces- Local Equivalence of surfaces and parameterized surfaces, Differential parameters, Isometric surfaces, Geodesic curvature, normal curvature, minimal surfaces.

Reference

MA7258: ADVANCED OPERATIONS RESEARCH

(Prerequisite: Knowledge of Linear programming)

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Total hours: 42

Module I: (12 hours)
Mathematical preliminaries, Maximum and Minimum-Quadratic forms-Gradient and Hessian matrices, Unimodal functions, Convex sets, Convex and concave functions, Mathematical programming Problems, Varieties and characteristics, Difficulties caused by nonlinearity, Role of convexity in Non linear programming, Unconstrained optimization, Search methods, Fibonacci search, Golden section search.

Module II: (10 hours)
Hooke and Jeeve’s Method, Optimal gradient method, Newton’s method- Constrained nonlinear optimization, Constrained optimization with equality constraints, Lagrangian method, Sufficiency conditions, Optimization with inequality constraints, Kuhn-Tucker conditions, Sufficiency Conditions.

Module III: (10 hours)
Quadratic programming, Separable programming-Frank and Wolfe’s method, Kelley’cutting plane method, Rosen’s gradient projection method, Fletcher-Reeve’s method, Penalty and Barrier method.

Module IV: (10 hours)
Integer linear programming, Gomory’s cutting plane method, Branch and Bound Algorithm, Travelling salesman’s problem, knapsack problem, Introduction to optimization software.

References:
MA7259: ADVANCED TOPOLOGY

(Pre-requisite: Topology)

Total hours: 56

Module I: (12 hours)


Module II: (15 hours)

Separation Axioms and Covering Properties - Separation axioms: Hausdorff, regular, Tychonoff, and normal topological spaces, Covering properties: Compactness, Lindelofness, paracompactness, metacompactness, Relations between covering properties and separation axiom, Normality of paracompactspaces, paracompactness of Lindel’of spaces, Preservation of separation and covering properties.

Module III: (15 hours)

Metrizibility and Connectedness - The metrization theorems of Urysohn and Bing, Smirnov, Nagata. Connectedness and total disconnectedness: Definitions and examples of connectedness, total disconnectedness, zero-dimensionality. Local properties: Local compactness, local connectedness

Module IV: (14 hours)


References

MA7260: SIMULATION AND MODELLING

(Pre-requisite: Knowledge of Statistical Methods)

Total hours: 42

Module I: (12 hours)

Introduction to system simulation, Introduction: Systems and models, Computer simulation and its applications, Continuous system simulation, Modelling continuous systems, simulation of continuous systems, Discrete system simulation- Methodology, event scheduling and process interaction approaches, Random number generation, testing of randomness, generation of stochastic variates, Random samples from continuous distributions, Uniform distribution, Exponential distribution, m-Erlang distribution, Gamma distribution, Normal distribution, Beta distribution, Random samples from discrete distributions – Bernoulli, Discrete uniform, Binomial, Geometric and Poisson.

Module II: (10 hours)

Evaluation of Simulation Experiments and Simulation Languages, Evaluation of simulation experiments, verification and validation of simulation experiments, Statistical reliability in evaluating simulation experiments, Confidence intervals for terminating simulation runs, Simulation Languages: Programming Considerations, General features of GPSS, SIMSCRIPT and SIMULA.

Module III: (10 hours)


Module IV: (10 hours)


References

MA7261: MULTI-VARIATE STATISTICAL ANALYSIS

(Pre-requisite: Knowledge of Statistical Methods)

Total hours: 42

Module I: (10 hours)

Random vectors and matrices, Random vectors and matrices, mean vectors and covariance matrices, matrix inequalities and maximization, Sample Geometry and random sampling-geometry of the sample, random samples and expected values of the sample mean and covariance matrix, generalized variance, sample mean, covariance and correlation as matrix operations, sample values of linear combinations of variables.

Module II: (11 hours)

The multivariate normal distribution, the multivariate normal distribution: Introduction, the multivariate normal density and its properties, sampling from a multivariate normal distribution and maximum likelihood estimation, the sampling distribution of $\bar{X}$ and $S$, Large-Sample behaviour of $\bar{X}$ and $S$, assessing the assumption of normality transformations to near normality.

Module III: (11 hours)

Inferences about a mean vector: Introduction, the plausibility of $\mu_0$ as a value for a normal population mean, Hotelling’s $T^2$ and likelihood ratio tests, confidence regions and simultaneous comparisons of component means, Large-sample inferences about population mean vector, inferences about mean vectors when some observations are missing.

Module IV: (10 hours)

Comparisons of several multivariate means: Introduction paired comparison and a repeated measures design, comparing mean vectors from two populations, comparison of several multivariate population means (one-way MANOVA), simultaneous confidence intervals for treatment effects, two-way multivariate analysis of variance.

References

MA7262: DECISION THEORY

(Pre-requisite: Statistical Methods)

Total hours: 56

Module I: (14 hours)

Game theory and decision theory: Basic elements, a comparison of game theory and decision theory, Decision functions, Optional decision rules, Baye’s rule for estimation problems.

Module II: (14 hours)

The main theorems of decision theory: Admissibility and completeness, decision theory, admissibility of Baye’s rules, basic assumptions, existence of Baye’s decision rules, existence of minimal complete class, the separating hyper plane theorem, essential completeness of the class of nonrandomized decision rules, the minimax theorem, the complete class theorem, solving for minimax rules.

Module III: (14 hours)

Sufficient statistics and risk functions: The multivariate Normal distribution, sufficient statistics, essentially complete classes of rules based sufficient statistics, complete sufficient statistics, and continuity of the risk functions.

Module IV: (14 hours)

Decision problems: Invariant statistical decision problems, admissible and minimax invariant rules, location and scale parameters, minimax estimates for the parameters of a distribution function. Multiple decision problem- monotone multiple decision problem, Baye’s rules in multiple decision problems.

References

MA7263: REGRESSION ANALYSIS

(Pre-requisite: Knowledge of Statistical Methods)

Total hours: 56

Module I: (14 hours)

Simple regression with one independent variable(X), assumptions, estimation of parameters, standard error of estimator, testing of hypothesis about regression parameters, standard error of prediction, Testing of hypotheses about parallelism, equality of intercepts, congruence. Extrapolation, optimal choice of X. Diagnostic checks and correction: graphical techniques, tests for normality, uncorrelatedness, homoscedasticity, lack of fit, modifications like polynomial regression, transformations on Y or X, WLS, inverse regression X(Y).

Module II: (14 hours)

Multiple regression: Standard Gauss Markov Setup, Least square(LS) estimation, Error and estimation spaces, Variance- Covariance of LS estimators, estimation of error variance, case with correlated observations, LS estimation with restriction on parameters, Simultaneous estimation of linear parametric functions, Test of Hypotheses for one and more than one linear parametric functions, confidence intervals and regions, ANOVA.

Module III: (14 hours)

Non Linear regression (NLS) : Linearization transforms, their use & limitations, examination of non linearity, initial estimates, iterative procedures for NLS, grid search, Newton- Raphson, steepest descent, Marquardt’s methods.

Module IV: (14 hours)

Logistic Regression: Logit transform, ML estimation. Tests of hypotheses, Wald test, LR test, score test, test for overall regression, multiple logistic regression, forward and backward method, interpretation of parameters relation with categorical data analysis, generalized linear model: link functions such as Poisson, binomial, inverse binomial, inverse Gaussian, gamma.

References

MA7264: ALGEBRAIC TOPOLOGY

(Pre-requisite: Knowledge of Topology)

Total hours: 56

Module I: (14 hours)


Module II: (14 hours)

Fundamental group: Homotopy, homotopy classes, Fundamental group, Change of base point, Topological invariance.

Module III: (14 hours)

Homology groups: Geometrical motivation, Euclidean simplexes, Linear mappings, singular simplexes, chains, Boundary of a simplex, Boundaries and cycles on any space, Homologous cycles and homology groups, Relative homology.

Module IV: (14 hours)

Induced Homomorphisms, Topological invariance of homology groups, Homotopic mappings and the homology groups, Prisms, Homology sequences, Simplical complexes.

References

MA7265: STATISTICAL DIGITAL SIGNAL PROCESSING

(Pre-requisite: Knowledge of Statistical methods)

Total hours: 56

Module I: (15 hours)

Module II: (12 hours)

Module III: (14 hours)

Module IV: (15 hours)

References

MA7266: STATISTICAL METHODS FOR QUALITY MANAGEMENT

(Pre-requisite: Knowledge of Statistical Methods)

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Total Hours: 42

**Module I: (11 hours)**

**Module II: (11 hours)**
Statistical process control: Chance and assignable causes of quality variation, setting up of operating control charts for $\bar{X}$ and $R$, Control charts for $\bar{X}$ and $S$, Control charts for individual measurements, Applications of variables control charts. Control charts for Attributes- control charts for Fraction nonconforming, control charts for nonconformities (defects).

**Module III: (10 hours)**
Cumulative sum and exponentially weighted moving average control charts- The cumulative-sum control charts, The exponentially weighted moving-average control charts, the moving average moving control charts. Statistical process control techniques, process capability analysis, acceptance sampling for attributes.

**Module IV: (10 hours)**
Reliability Statistics: Reliability definition, availability, reliability bathtub curve, estimating MTBF, reliability prediction, confidence interval for MTBF, testing, system reliability, series systems, parallel systems, Baye’s theorem applications, non-parametric and related test designs, hazard function, Weibul distribution, Log-normal distribution, stress- strength inference, Binomial confidence intervals, Arrhenius model, sequential testing.

**References:**
MA7267: GENERALISED SET THEORY

(Pre-requisite: Nil)

Total hours: 56

Module I (14 hours)

An overview of basic operations on Fuzzy sets and Multisets, Multiset relations, Compositions, equivalence multiset relations and partitions of multisets, Multiset functions, Fuzzy Multisets.

Module II (14 hours)

Rough sets, Approximations of a set, Properties of Approximations, Rough membership function, Rough sets and Reasoning from data: Information systems, Decision tables, Dependency of attributes, Reduction of attributes, Indiscernibility matrices and functions.

Module III (14 hours)

Soft sets, Tabular representation of a soft set, Operations with Soft sets: soft subset, complement of a soft set, null and absolute soft sets, AND and OR operations, Union and intersection of soft sets, DeMorgan laws, Applications and soft analysis.

Module IV (14 hours)

Fuzzy soft sets, Operations on fuzzy soft sets, Soft fuzzy sets and its properties, Fuzzy rough sets and rough fuzzy sets, Rough multisets, Genuine sets, Applications.

References:

MA7268: FOURIER ANALYSIS

(Pre-requisite: Nil)

Total hours: 56

Module I (14 hours)


Module II: I (14 hours)

\( L^p \) spaces, Convolution of functions, Young’s inequality, approximate identity, Regularisation of functions, Pointwise convergence, Fourier transform in \( L^1(R) \), Riemann-Lebesgue lemma, Multiplication formula, Inversion, Translations and dilations, Multiplication and differentiation.

Module III: I (14 hours)

Abel means and Poisson kernel, Uniqueness theorem for Fourier transform in \( L^1(R) \), Fourier transform in \( L^2(R) \), Multiplication formula, Plancherel theorem, Uniqueness theorem in \( L^2(R) \), Harmonic functions, Dirichlet problem for the upper half plane, Point wise and norm convergence, Dirichlet problem for the disc.

Module IV: I (14 hours)

Eigen functions of Fourier transform, Gaussian – Hermite functions, Schwartz space, Paley-Wiener space Paley-Wiener theorem, Uncertainty principle, Hardy classes, Hardy’s theorem.

References:

MA7269 FUZZY SET THEORY AND APPLICATIONS

(Pre-requisite: Nil)

Total hours: 42

Module I: (11 hours)
Introduction, crisp sets an overview, the notion of fuzzy sets, basic concepts of fuzzy sets, membership functions, methods of generating membership functions, defuzzification methods, operations on fuzzy sets, fuzzy complement, fuzzy union, fuzzy intersection, combinations of operations, General aggregation operations.

Module II: (11 hours)
Fuzzy numbers, arithmetic operations on intervals, arithmetic operations on fuzzy numbers, fuzzy equations, crisp and fuzzy relations, binary relations, binary relations on a single set, equivalence and similarity relations, compatibility or tolerance relations.

Module III: (10 hours)
Fuzzy measures, belief and plausibility measures, probability measures, possibility and necessity measures, possibility distribution, relationship among classes of fuzzy measures.

Module IV: (10 hours)
Classical logic: an overview, fuzzy logic, approximate reasoning, other forms of implication operations, other forms of the composition operations, fuzzy decision making, fuzzy logic in database and information systems, fuzzy pattern recognition, fuzzy control systems.

References:
MA7270: STOCHASTIC PROCESSES

(Pre-requisite: Knowledge of Measure and Probability)

Total hours: 56

Module I: (14 hours)

Elements of stochastic processes, Classification of general stochastic processes, Markov Chains: Definition, examples, transition probability matrix, classification of states, basic limit theorem, limiting distribution of Markov Chains.

Module II: (14 hours)

Continuous time Markov Chains: General pure birth processes and Poisson processes, more about Poisson processes, A counter model, Birth and Death processes with absorbing states, Finite state continuous time Markov Chains.

Module III: (14 hours)

Renewal Processes: Definition of a renewal process and related concepts, examples of renewal processes, special renewal processes, renewal equation and elementary renewal theorem, the renewal theorem, generalizations and variations on renewal processes, applications of renewal theory.

Module IV: (14 hours)


References:

MA7271: CODING THEORY

(Pre-requisite: Nil)

Total hours: 56

MODULE 1: (14 hours)

Introduction to Coding Theory: Basic assumptions, Correcting and detecting error patterns, Information rate, The effects of error correction and detection, Finding the most likely codeword transmitted, Some basic algebra, Weight and distance, Maximum likelihood decoding, Reliability of MLD, Error-detecting codes.

MODULE 11: (14 hours)

Linear Codes: Two important subspaces, Independence, basis, dimension, Matrices- Bases for $C = \langle S \rangle$ and $C^\perp$ - Generating matrices and encoding, Parity-check matrices, Equivalent codes, Distance of a linear code, Cosets, MLD for linear codes, Reliability of IMLD for linear codes.

MODULE 111: (14 hours)

Perfect and Related Codes: Some bounds for codes, Perfect codes, Hamming codes, Extended codes, The extended Golay code, Decoding the extended Golay code, The Golay code, Reed-Muller codes- Fast decoding for $RM(1,m)$. 

MODULE 1IV: (14 hours)

Cyclic Linear Codes: Generating and parity check matrices for cyclic codes, Finding cyclic codes, Dual cyclic codes, BCH Codes, Reed–Solomen Codes.

References

MA7272: Reliability of Systems

(Pre-requisite: Nil)

Total hours: 56

Module I: (14 hours)

Introduction to reliability, Basic concepts, Cut sets, Path sets, Minimal cut and path sets, Bounds for reliability, Reliability and Quality, Maintainability and Availability, Reliability analysis, Causes of failures, Catastrophic and Degradation failures, Useful life of components, Component reliability and hazard models, Mean time to failure, system reliability models, System with components in series, parallel, k/n systems, System with mixed mode failures.

Module II: (14 hours)

Redundancy Techniques, Component v/s unit redundancy, Weakest link techniques, Mixed redundancy, Stand by redundancy, Redundancy optimization, Double failure and redundancy, Maintainability and availability concepts, Two unit parallel system with repair, Signal redundancy, Time redundancy, Software redundancy.

Module III: (14 hours)

Hierarchical systems, Path determination method, Boolean Algebra method, Cut set approach, Logic diagram approach, Conditional probability approach, System cost and reliability approximations, Economics of reliability engineering, Economic cost, manufacturing cost, customers cost, Reliability achievement cost models, Depreciation cost models, Reliability management, Management policy and decisions.

Module IV: (14 hours)

Life testing: Introduction, hazard rate functions, Exponential distribution in life testing, Simultaneous testing-stopping at r-th failure, Stopping by fixed time, sequential testing, Accelerated testing, Equipment Acceptance testing, Software reliability, Software reliability models, Reliability Allocation, A two sample problem.

References

MA7273: OPERATOR THEORY

(Pre-requisite: Knowledge of Functional analysis)

Total hours: 42

Module I: (10 hours)

Banach algebras, Gelfand theory, C*- algebras the GNS construction, spectral theorem for normal operators, Fredholm operators and its properties, semi-Fredholm operators, product of operators.

Module II: (11 hours)


Module III: (11 hours)


Module IV: (10 hours)

Unbounded operators: Basic theory of unbounded self-adjoint operators, unbounded Fredholm operators and its properties, essential spectrum, unbounded semi-Fredholm operators, Spectral theorem for an unbounded self adjoint operators.

References

MA7274: WAVELETS THEORY

(Pre-requisite: Nil)

Total hours: 42

Module I: (11 hours)


Module II: (11 hours)

Construction of wavelets on $\mathbb{Z}_N$, The Haar system, Shannon Wavelets, Real Shannon wavelets, Daubechies’s D_6 wavelets on $\mathbb{Z}_N$, Examples and applications.

Module III: (11 hours)

Wavelets on $\mathbb{Z}$: $\ell^2(\mathbb{Z})$, Complete orthonormal sets in Hilbert spaces, $L^2(-\pi, \pi)$ and Fourier series, The Fourier Transform and convolution on $\ell^2(\mathbb{Z})$, First stage Wavelets on $\mathbb{Z}$, Implementation and Examples.

Module IV: (9 hours)

Wavelets on $\mathbb{R}$: $L^2(\mathbb{R})$ and approximate identities, The Fourier transform on $\mathbb{R}$, Multiresolution analysis, Construction of MRA.

References:

MA7275: QUEUEING THEORY

(Pre-requisite: knowledge of Probability and Measure & Stochastic Processes)

Total hours: 42

Module I: (11 hours)

Simple Markovian Queuing Models – Introduction to queuing theory, Characteristics of queuing processes, Measures of effectiveness, Markovian queueing models, steady state solution of the M/M/1 model, waiting time distributions, Little’s formula, queues with parallel channels and truncation, Erlang’s loss formula, Queues with unlimited service, finite source queues.

Module II: (12 hours)

Transient behaviour of M/M/1 queues, transient behaviour of M/M/\infty, Busy period analysis for M/M/1 and M/M/c models. Advanced Markovian models, Bulk input \(M^{[Y]} / M / 1\) model, Bulk service \(M / M^{[Y]} / 1\) model, Erlangian models \(M / E_a / 1\) and \(E_a / M / 1\), A brief discussion of priority queues.

Module III: (9 hours)


Module IV: (10 hours)

Models with general arrival pattern, The M/G/1 queuing model, The Pollaczek-khintchine formula, Departure point steady state system size probabilities, ergodic theory, special cases \(M / E_a / 1\) and M/D/1, waiting times, busy period analysis, general input and exponential service models, arrival point steady state system size probabilities.

References

MA7276: NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

(Pre-requisite: Nil)

Total hours: 56

Module 1: (14 hours)

Introduction; Initial Value problem for first order ODEs, Initial Value problem for system of first order ODEs’, Reduction of higher order ODEs to first order systems; First order system with constant coefficients, linear difference equation with constant coefficients,

Module 2: (14 hours)


Module 3 :(14 hours)

General Linear Multi-Step Methods, derivation through Taylor expansions, derivation through numerical integration, derivation through interpolation, convergence, order and error constants, Local and global truncation error, consistency and zero stability, Error bounds and local and global truncation error, weak stability theory, interval of absolute and relative stability, comparison of implicit and explicit Linear Multistep methods,

Module 4:(14 hours)


References

MA7277: Numerical solutions for partial differential equations

(Prerequisite: Knowledge of programming in Matlab)

Total hours: 56

Module-1: (13 hours)


Module 2: (14 hours)

Parabolic equations: explicit and implicit methods for one and two dimentional parabolic equations, Crank-Nicolson method, numerical examples, weighted average approximation, consistency, convergence and stability, alternate direction method in two dimensions, Peaceman-Rachford scheme, Douglas-Rachford scheme.

Module 3: (15 hours)

Hyperbolic equations: Finite difference methods for first and second order wave equation, Lax-wendroff explicit method, CFL condition for one and two dimensionss, ADI schemes fo two dimensional hyperbolic equations, Lax-wendroff method for a system of hyperbolic equations, Wendroff’s implicit approximation, reduction of a first order equation to a system of ordinary differential equations, numerical examples.

Module 4: (14 hours)

Elliptic equations: Numerical examples: a torsion problem, a heat conduction problem with derivative boundary conditions. Finite differences in polar co-ordinates, techniques near a curved boundary, improvement of the accuracy of the solutions. Analysis of the discretization error of the five-point approximation to Poisson’s equation.

(Most of the tutorial classes will be dealt with implementation of numerical schemes for solving partial differential equations in Matlab.)

References:

MA7278: SPECTRAL THEORY OF HILBERT SPACE OPERATORS

(Pre-requisite: nil)

Total hours: 56

Module I: (13 hours)

Elements of Hilbert space theory, Bounded linear operators on Hilbert spaces, Bounded linear functionals, projection , Riesz representation theorem, Adjoint, Self adjoint, Unitary Normal operators.

Module II: (10 hours)

Spectral properties of bounded linear operators, Resolvant and spectrum, spectral theory, Complex analysis in spectral theory.

Module III: (16 hours)

Compact linear operators, spectral theory of compact self adjoint operators; Formula for the inverse operator, Minimum-maximum Properties of eigenvalues, compact normal operators, Operator equations, Fredholm alternative.

Module IV: (17 hours)

Spectral properties of bounded self adjoint linear operators, Positive operators, Square root of an operator, Projection operators, spectral family and spectral family of bounded self adjoint linear operator, spectral representation of bounded self adjoint Linear operators, Extension of spectral theorem to continuous functions.

References

MA7279: NUMERICAL LINEAR ALGEBRA

(Pre-requisite: nil)

Total Hours: 56

Module I: (14 hours)

Review Linear Algebra Basic Concepts, Conditioning and Stability; Condition numbers, Floating point arithmetic, Stability of various algorithms, Linear Equation Solving: Gaussian Elimination, Pivoting, Stability of Gaussian Elimination, Cholesky Factorization, Jordan canonical form and applications.

Module II: (17 hours)


Module III: (13 hours)

Singular Value Decomposition (SVD), Computing the SVD, applications, QR algorithm for SVD

Module IV: (12 hours)

Generalized inverses of matrices, computing the Moore-Penrose generalized inverse of a matrix.

References

MA7280: FRACTAL THEORY AND APPLICATIONS

(Pre-requisite: nil)

Total hours: 56

Module I: (14 hours)

The space of fractals and Iterative function systems: The matrix space \((H(X),h)\), the completeness of space of fractals, transformations on the real line, affine transformations in the Euclidean plane, Mobius transformations on the Riemann sphere, Analytic transformations, the contraction mapping theorem, the deterministic algorithm, random iteration algorithm, condensation sets, the continuous dependence of fractals on parameters.

Module II: (14 hours)

Chaotic dynamics on fractals and fractal dimension: The addresses of points on fractals, continuous transformations from code space to fractals, dynamical systems, dynamics on fractals, equivalent dynamical systems, shadow of deterministic dynamics, shadowing theorem, Chaotic dynamics on fractals, fractal dimension, theoretical and experimental determination of fractal dimension, Hausdorff-Besicovitch dimension.

Module III: (14 hours)

Fractal interpolation,Julia sets and Mandelbrot’s sets: Applications for fractal functions, fractal interpolation functions, the fractal dimension of fractal interpolation functions, hidden variable fractal interpolation, space filling curves, escape time algorithm, Julia sets, IFS for Julia sets, Application of Julia sets to Newton’s method Invariant sets of continuous open mappings, map of fractals, Mandelbrot’s sets, Mandelbrot’s sets for Julia sets.

Module IV: (14 hours)

Measures on fractals and application: Invariant measures on fractals, measures, integration, Elton’s theorem, recurrent iterated function systems, applications to computer graphics, fractal compression, fractal antennas.

References