MULTI-PERIOD MODEL FOR PART FAMILY/MACHINE CELL FORMATION

- Given a set of parts, processing requirements, and available resources
  - The objective of the part family/machine cell (PF/MC) formation problem is to obtain a satisfactory partition of parts into families and machines into cells
  - The resulting system performs well with respect to the design objectives
- Effectiveness of a cellular manufacturing system is sensitive to fluctuations in product demand and product mix
- Generally PF/MC formation models assume that both of these factors remain constant
- In reality, the demand for a product varies over its life cycle, new products are introduced, and the production of older products are discontinued

**Objectives included in the multi-period formulation**
- Minimize intercell transfers of parts
- Minimize duplication of machines
- Minimize between period reconfiguration of cells
- Intercell transfers decrease the efficiency of the cellular system by increasing material handling requirement, lengthening flow lines, and complicating production control
- The minimization of intercell transfers and the minimization of machine duplication are conflicting
- Tradeoff between intercell transfers and machine duplication are allowed by including both objectives
- Minimize system re-configuration is particular to the multi-period formulation
- System reconfiguration is defined as changing the composition of the machine cells by moving machine from one cell to another in subsequent periods
- By reconfiguring the machine cells, the cellular system can continue to operate efficiently as the product mix and demand levels change
- Reconfiguration is not without drawback – moving machines from cell to cell requires effort and can lead to the disruption of production
- This suggests for a tradeoffs between system reconfiguration and increased intercell transfers and/or increased machine duplication
- To accommodate all these objectives and to formalize the tradeoffs being made, system performance with respect to each objective is measured in monetary terms
- In cost terminology the objectives are re-written as:
  - Minimize intercell material handling costs
  - Minimize capital investment in machines, and
Minimize machine relocation costs

The aggregated objective of the multi-period PF/MC formation problem is to minimize the system cost

**Mixed-integer programming formulation of the multi-period PF/MC formation**

$i$ – index of parts, $i = 1,2,\ldots,N$

$j$ – index of machines, $j = 1,2,\ldots,M$

$k$ – index of cells, $k = 1,2,\ldots,C$

$l$ – index of periods, $l = 1,2,\ldots,p$

System parameters:

$D_{il}$ - Demand (production volume) for part $i$ in period $l$

$S_{il}$ - Number of processing operations for part $i$ in period $l$

$O(i,r,l)$ - Machine type required by the $r$th operation on part $i$ in period $l$

$T_{ijl}$ - Processing time of part $i$ on machine type $j$ in period $l$

$M_j$ - Number of type $j$ machines available at start of planning horizon

$C_j$ - Capacity of machine type $j$

$P_{jl}$ - Cost of acquiring a type $j$ machine in period $l$

$H_{il}$ - Intercell per unit material handling cost for part $i$ in period $l$

$R_{jl}$ - Cost of relocating machine type in period $l$

$LM$ – Minimum number of machines per cell

$LP$ – Minimum number of parts per family

$A$ – a large number

Decision Variables

$$x_{iki} = \begin{cases} 
1 & \text{if part } i \text{ is assigned to cell } k \text{ during period } l \\
0 & \text{Otherwise} 
\end{cases}$$
\[ y_{jkl} = \begin{cases} 1 & \text{if machine } j \text{ is assigned to cell } k \text{ during period } l \\ 0 & \text{Otherwise} \end{cases} \]

\[ n_{jkl} \] - Number of type \( j \) machines assigned to cell \( k \) during period \( l \)

Notations to simplify the expression of objective function

\( q_{il} \) - Number of intercell transfers that occur during the production of part \( i \) during period \( l \)

\( b_{jl} \) - Number of additional type \( j \) machines acquired at the beginning of period \( l \)

\( u_{jl} \) - Number of type \( j \) machines that are relocated between period \((l-1)\) and period \( l \)

Objective Function

Minimize

\[
Z = \sum_{l=1}^{p} \left[ \sum_{i=1}^{N} (H_{il} \times D_{il} \times q_{il}) + \sum_{j=1}^{M} (P_{jl} \times b_{jl}) + \sum_{j=1}^{M} (R_{jl} \times u_{jl}) \right] \quad \ldots[1]
\]

Where

\[
q_{il} = \sum_{k=1}^{C} x_{ikl} \left[ \sum_{r=1}^{S_{il}} (1 - y_{O(i,r,l)kl} \times y_{O(i,r+1,l)kl}) \right] \forall i, l \quad \ldots[2]
\]

\[
b_{jl} = \max \left\{ 0, \sum_{k=1}^{C} n_{jkl} - M_{j} - \sum_{s=1}^{l-1} b_{js} \right\} \forall j, l \quad \ldots[3]
\]

\[
u_{jl} = \sum_{k=1}^{C} \left[ \max \left\{ 0, n_{jkl} - n_{jk(l-1)} \right\} \right] - b_{jl} \forall j, l \quad \ldots[4]
\]

Subjected to:

Constraints to ensure that each part is assigned only to a cell for periods in which demand exists for the part

\[
\sum_{k=1}^{C} x_{ikl} = 1 \quad \forall i, l \quad \ldots[5]
\]
Within-cell capacity constraints

\[ \sum_{i=1}^{N} D_{ij} T_{ijl} x_{ikl} y_{jkl} \leq C_j n_{jkl} \quad \forall j,k,l \quad \ldots[6] \]

Entire system capacity constraints

\[ \sum_{i=1}^{N} D_{ij} T_{ijl} \leq C_j \sum_{k=1}^{c} n_{jkl} \quad \forall j,l \quad \ldots[7] \]

Lower limits on the number of machines per cell

\[ \sum_{j=1}^{M} y_{jkl} \geq L M \quad \forall k,l \quad \ldots[8] \]

Lower limits on the number of parts per cell

\[ \sum_{i=1}^{N} x_{ikl} \geq L P \quad \forall k,l \quad \ldots[9] \]

Constraints to ensure that the number of units of a given machine type in a cell is equal to zero unless the machine has been assigned to the cell

\[ n_{jkl} \leq A y_{jkl} \quad \forall j,k,l \quad \ldots[10] \]

Integrality constraints

\[ x_{ikl}, y_{jkl} = \{0, 1\} \quad \forall i, j, k, l \quad \ldots[11] \]

\[ n_{jkl} \geq 0, \text{integer} \quad \forall i, j, k, l \quad \ldots[12] \]

**QUESTIONS:**

1. What is a multi-period cell formation problem? Write down the objective function of multi-period cell formation. Use appropriate notation for the variables used and identify the decision variables of the multi-period cell formation problem.

2. What are the costs generally considered in the objective function of multi-period cell formation? Describe the costs involved and the tradeoff while forming an objective function in multi-period cell formation.

3. Describe an objective function that is suitable for a cellular manufacturing system design that considers product mix and demand variation from period to period in a planning horizon. Explain the kind of tradeoff involved in the formulation.
4. A production system has the following details: Number of machine types = 6, Number of parts = 10. The resource data and part data for the production system are given in the tables below.

PF/MC formation requirements: Number of cells = 3, Minimum number of machine types in each cell = 2, Minimum number of parts in each cell = 2.

The planning horizon contains 2 periods.

Table 1. Resource data for the problem

<table>
<thead>
<tr>
<th>Machine type</th>
<th>Acquisition cost</th>
<th>Relocation cost</th>
<th>Capacity (time units/period)</th>
<th>Number available at period 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22000</td>
<td>1000</td>
<td>8000</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>26000</td>
<td>3000</td>
<td>12000</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>22500</td>
<td>1250</td>
<td>13000</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>26000</td>
<td>3000</td>
<td>9000</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>23000</td>
<td>1200</td>
<td>10000</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>25000</td>
<td>2500</td>
<td>7000</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2. Part data for the problem

<table>
<thead>
<tr>
<th>Part number</th>
<th>M/H* cost per unit</th>
<th>Operation sequence machine (processing time)</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Period 1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>6 (5)-4(4)-5 (5)-1(4)-3(4)-2(4)</td>
<td>700</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4(5)- 2 (4)- 3(5)-1(4)</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>5 (5)-3(4)-6(3)-1(6)-2(5)-4(4)</td>
<td>750</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>2(6)-3 (6)-1(4)-4(5)-5(4)</td>
<td>400</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>3 (5)-1 (6)-6(3)-4(4)- 5 (6)-2(5)</td>
<td>300</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>5(4)-1(4)-4(3) -2(6)-6(5)- 3(4)</td>
<td>300</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>1(5)-4(4)-3(4)-2(3)</td>
<td>400</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>3 (4)-5(5)-4(4)-2(5)-1(4)</td>
<td>300</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>4(5)- 2 (6)-1(4)-3(6)</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>2(4)-3(5)-4(4)-1(6)</td>
<td>700</td>
</tr>
</tbody>
</table>

* M/H – Material Handling

(i) Develop a mathematical programming formulation for PF/MC formation for this problem.
Write down a feasible solution and calculate the objective function value for the solution identified.

5. A production system has the following details: Number of machine types = 6, Number of parts = 9, Intercell material handling cost = Rs 1 per unit per transfer.

<table>
<thead>
<tr>
<th>Machine type</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity (time units)</td>
<td>8000</td>
<td>12000</td>
<td>13000</td>
<td>9000</td>
<td>10000</td>
<td>7000</td>
</tr>
</tbody>
</table>

The management decided to have cellular manufacturing system with the following details.
Number of cells = 3, Minimum number of machine types in each cell = 2, Maximum number of machine type in a cell = 3, and Minimum number of parts in each cell = 2.

A typical solution for the problem is as follows:

C1 = {1, 4, 5}, PF1 = {4, 6, 7, 8, 9}
C2 = {2, 3, 4}, PF2 = {2, 3}
C3 = {4, 5, 6}, PF3 = {5, 1}
C – stands for cell and PF – stands for part family.

(a) What is the intercell material handling cost for above solution? How many units of machine 1 should be present in cell 1?

(b) If a mathematical programme is used to define cell formation and part assignment and if a feasible solution (typical solution) given above is applicable, generate constraints that should ensure sufficient capacity for each machine type in a cell. Use the following notations for generating the constraints.
$i$ – index of parts, $i = 1,2,\ldots,N$

$j$ – index of machines, $j = 1,2,\ldots,M$

$k$ – index of cells, $k = 1,2,\ldots,C$

$D_i$ - Demand (production volume) for part $i$

$T_{ij}$ - Processing time of part $i$ on machine type $j$

$C_j$ - Capacity of machine type $j$

$x_{ik} = \begin{cases} 
1 & \text{if part } i \text{ is assigned to cell } k \\
0 & \text{Otherwise}
\end{cases}$

$y_{jkl} = \begin{cases} 
1 & \text{if machine } j \text{ is assigned to cell } k \text{ during period } l \\
0 & \text{Otherwise}
\end{cases}$

$n_{jkl}$ - Number of type $j$ machines assigned to cell $k$

(c) If a mathematical programme is used to define cell formation and part assignment and if the feasible cells are $C_1 = \{1, 4\}$, $C_2 = \{2, 3\}$ and $C_3 = \{5, 6\}$ then define a decision variable for the problem which shows a part assignment to a cell (part family). A part can be assigned to a part family only. Generate all such constraints for the given problem.