

#### ABOUT THE AUTHOR

Dr. K P Mohandas is currently Professor in Electrical Engineering and Dean (Post Graduate Studies and Research) in National Institute of Technology (formerly Regional Engineering College) Calicut. He has graduated from Kerala University in 1968 with first class distinction and first rank and has secured his M.Tech degree in Control Systems from IIT Madras and PhD Degree from IIT Delhi in Systems Theory. He has been teaching Control and Systems Theory for the last three decades in India and abroad at undergraduate and post graduate level and had been active in research as shown in the large number of publications he has in several International journals and conferences. He is with REC (NIT), Calicut since 1969 and had been working abroad on leave as Visiting Professor in Data Storage Institute, National University, Singapore, Professor of Electrical Engineering in Cukurova University, Adana, Turkey, and Professor and Head of the Department of Electrical and Electronics Engineering in European University of Lefke, Republic of North Cyprus. In addition to his research publications, he has published several popular science articles in periodicals and authored two books in Malayalam published by the State Institute of Languages, Kerala. He was one of the editors of the Proceedings in the National Systems Conference 1998 published by M/s Allied Publishers, New Delhi.

#### ABOUT THE BOOK

The book is divided into ten chapters with the first chapter being a very brief introduction to classical control theory. The second chapter gives the classical design techniques using Bode plots and root locus technique. Analysis of discrete time systems is presented in Chapter 3 using z-transforms. Chapters 4, 5 and 6 deal with state space modelling, solution of state equation and design of control systems using state space model with a glimpse on the design of observers, and state feed back controllers. Chapters 7 and 8 deal with nonlinear systems, the former on phase plane analysis and the latter on describing function method. Even though both these methods were developed long time back, these methods are still useful to get some insight into the behaviour of nonlinear systems. Chapter 9 discusses in depth the Lyapunov's method for stability analysis of systems and Chapter 10 is a brief introduction to concepts and methods of optimal control. Several worked examples and a summary—'points to remember' have been added in each chapter. A set of multiple choice questions has been added at the end of the book which is useful for students in the preparation for objective type tests and Answers to additional numerical problems have been provided. An introduction to the MATLAB software package is given in Appendix.

**SANGUINE**

sanguine\_publishers@vsnl.net

ISBN 81 88849 26 X



9 798188 849269



SANGUINE

Spine

DR. K.P. MOHANDAS

MODERN CONTROL ENGINEERING

Spine

# MODERN CONTROL ENGINEERING



Dr. K.P. MOHANDAS

# Modern Control Engineering

*Dr. K. P. Mohandas*

Department of Electrical Engineering,  
National Institute of Technology,  
NIT Campus PO, Calicut



SANGUINE

Sanguine Technical Publishers  
Bangalore 560 016, India  
2006

# Contents

## Chapter 1

Review of Classical Control Theory . . . . .	1
1.1. Introduction . . . . .	1
1.2. Transfer Function Model . . . . .	1
1.3. Time Response Analysis . . . . .	3
1.3.1. Test Inputs for Time Response Analysis . . . . .	3
1.4. Performance Specifications in Time Domain . . . . .	7
1.5. Time Response of First Order Systems . . . . .	9
1.6. Time Response of Second Order Systems . . . . .	11
1.7. Response of Higher Order Systems . . . . .	14
1.8. Frequency Domain Analysis . . . . .	15
1.8.1. Root Locus Technique . . . . .	15
1.8.2. Frequency Response . . . . .	16
1.8.3. Frequency Domain Performance Measures . . . . .	22
1.8.4. Determination of Frequency Domain Measures . . . . .	25
1.9. Correlation Between Time Domain and Frequency Domain Specifications	27
1.9.1. Peak Resonance, Resonant Frequency and Bandwidth in terms of $\zeta$ and $\omega_n$ . . . . .	27
1.9.2. Bandwidth . . . . .	28
1.10. Stability of Linear Systems . . . . .	29
1.10.1. Bounded Input Bounded Output Stability . . . . .	29

**viii Contents**

1.10.2. Characteristic Equation Roots and Impulse Response . . . . .	29
1.10.3. Stability from Polar Plot and Bode Plot . . . . .	29
1.10.4. Determination of Stability—Routh Hurwitz Criterion . . . . .	32

**Chapter 2**

Conventional Controllers and Classical Design . . . . .	35
2.1. Introduction . . . . .	35
2.2. Types of Controllers . . . . .	35
2.3. Compensating Networks . . . . .	38
2.3.1. Phase Lag Compensator . . . . .	43
2.3.2. Lead Lag Compensator . . . . .	44
2.4. Design of Lead Compensator Using Bode Plot . . . . .	46
2.5. Design of Lead Compensator Using Root Locus . . . . .	53
2.6. Design of Lag Compensator Using Bode Plot . . . . .	58
2.6.1. Lag Compensator Design Using Bode Plot . . . . .	58
2.7. Design of Lag Compensator Using Root Locus . . . . .	64
2.8. System Design Using PI Controllers . . . . .	68
2.9. Design of Lead-Lag Compensators . . . . .	71
2.9.1. Lead-Lag Design by Bode Plot Method . . . . .	71
2.9.2. Lead-Lag Compensation Using Root Locus . . . . .	74
2.10. PID Controller Design and Tuning . . . . .	77
2.10.1. Ziegler-Nichols Rules for Tuning PID Controllers . . . . .	77
2.11. Feedback Compensation . . . . .	81
2.12. Comparison of Different Types of Controllers . . . . .	84
2.13. Additional Worked Examples . . . . .	86
2.14. Points to Remember . . . . .	99
2.15. Exercise Problems . . . . .	101

**Chapter 3**

Discrete Data Control Systems . . . . .	103
3.1. Introduction . . . . .	103

3.2. Basic Elements of a Discrete Data System . . . . .	103
3.3. Advantages of Discrete Data Systems . . . . .	105
3.4. Different Types of Discrete Time Systems . . . . .	106
3.4.1. Pulse Amplitude Modulated Systems . . . . .	106
3.4.2. Pulse Width Modulated Systems . . . . .	106
3.4.3. Pulse Frequency Modulated (PFM) Systems . . . . .	107
3.5. Examples of Discrete Data Systems . . . . .	109
3.5.1. Stepper Motor Control of Hard Disk Drives . . . . .	109
3.5.2. A Simplified Single-Axis Autopilot Control System . . . . .	109
3.5.3. A Digital Computer Controlled Rolling Mill Regulating System . .	111
3.5.4. Computer Control of Turbine and Generator . . . . .	111
3.6. Sampling Process and Analysis . . . . .	112
3.6.1. Analysis of the Output of an Ideal Sampler in Time Domain . . . .	112
3.6.2. Analysis of the Output of a Practical Sampler in Frequency Domain	116
3.7. Sampling Theorem and Significance . . . . .	119
3.8. Data Reconstruction and Hold Circuits . . . . .	122
3.8.1. Zero Order Hold and Its Frequency Response . . . . .	123
3.8.2. First Order Hold and Data Reconstruction . . . . .	125
3.9. Mathematical Analysis of Discrete Data Systems . . . . .	128
3.9.1. $z$ -Transforms—Basic Definition . . . . .	128
3.9.2. $z$ -Transform Directly from Laplace Transform . . . . .	130
3.9.3. Example for Evaluation of $z$ -Transforms . . . . .	131
3.9.4. Properties of $z$ -Transforms . . . . .	133
3.9.5. Inverse $z$ -Transforms . . . . .	136
3.10. Solution of Difference Equation . . . . .	142
3.11. Pulse Transfer Function . . . . .	145
3.12. Block Diagram Reduction for Pulse Transfer Functions . . . . .	146
3.12.1. Cascaded Elements with Sampler in Between . . . . .	146
3.12.2. Cascaded Elements without Sampler in Between . . . . .	147
3.12.3. Closed Loop Control Systems . . . . .	148

**x Contents**

3.12.4. Zero-Order Hold and $G(s)$ in Cascade . . . . .	152
3.13. Time Response Analysis of Discrete Time Systems . . . . .	154
3.13.1. Time Response of Open Loop Systems . . . . .	154
3.13.2. Response of Closed Loop Systems . . . . .	155
3.14. Mapping Properties of Z-Transform . . . . .	157
3.15. Stability of Discrete Time Systems . . . . .	160
3.15.1. Routh-Hurwitz Criterion . . . . .	160
3.15.2. Jury's Test . . . . .	163
3.16. Response of Discrete Data Systems Between Sampling Instants . . . . .	165
3.17. Additional Worked Examples . . . . .	168
3.18. Points to Remember . . . . .	179
3.19. Exercise Problems . . . . .	182

**Chapter 4**

State Space Analysis of Systems . . . . .	185
4.1. Limitations of Classical Control . . . . .	185
4.2. Concept of State, State Space and State Variables . . . . .	186
4.3. State Model for Typical Systems . . . . .	188
4.3.1. Linear Systems . . . . .	188
4.3.2. Nonlinear Systems . . . . .	192
4.4. State Model of Linear Systems from Differential Equations . . . . .	193
4.5. State Variable Diagram and Block Diagram Representation of State Models	196
4.6. State Space Model for Physical Systems . . . . .	198
4.6.1. Electrical Circuits . . . . .	198
4.6.2. Mechanical Systems . . . . .	201
4.6.3. Electro-Mechanical Systems—DC Motors . . . . .	202
4.7. State Space Model from Transfer Functions . . . . .	205
4.7.1. Transfer Functions without Numerator Dynamics . . . . .	205
4.7.2. Transfer Function with Numerator Dynamics . . . . .	207
4.7.3. Series Decomposition . . . . .	211
4.7.4. Parallel Decomposition . . . . .	212

4.8. State Model for a Multi-Input Multi-Output System from Block Diagrams 216

4.9. Non-Uniqueness of State Space Model . . . . . 217

4.10. Transfer Function from State Model . . . . . 219

4.11. Canonical Models . . . . . 220

    4.11.1. Phase Variable Form or Controllable Canonical Model . . . . . 220

    4.11.2. Observable Canonical Model . . . . . 221

    4.11.3. Diagonal Canonical Model . . . . . 221

    4.11.4. Jordan Canonical Model . . . . . 222

4.12. Diagonalisation of a Matrix . . . . . 222

4.13. State Variable Description of Discrete Time Systems . . . . . 224

4.14. Additional Worked Examples . . . . . 228

4.15. Points to Remember . . . . . 238

4.16. Exercise Problems . . . . . 240

**Chapter 5**

Time Domain Analysis in State Space . . . . . 245

5.1. Solution of Time Invariant State Equation . . . . . 245

    5.1.1. Scalar Differential Equation—Homogeneous Case . . . . . 245

    5.1.2. Vector Matrix Differential Equation—Homogeneous Case . . . . . 246

    5.1.3. Laplace Transforms Method . . . . . 247

5.2. State Transition Matrix . . . . . 247

5.3. Solution of State Equation with Input . . . . . 249

5.4. Laplace Transform Approach . . . . . 251

5.5. State Transition Matrix from Cayleigh–Hamilton Theorem . . . . . 252

    5.5.1. Cayleigh–Hamilton Theorem . . . . . 253

    5.5.2. Computation of  $e^{At}$ —Method 1 . . . . . 253

    5.5.3. Computation of  $e^{At}$ —Method 2 . . . . . 254

5.6. Solution of Linear Time Varying State Equation . . . . . 258

5.7. Solution of State Equation in Canonical Forms . . . . . 260

    5.7.1. Diagonal Canonical Form and State Transition Matrix . . . . . 260

    5.7.2. Jordan Canonical Form . . . . . 261

**xii Contents**

5.8. Solution of Time Invariant Discrete Time State Equation . . . . .	262
5.9. Additional Worked Examples . . . . .	263
5.10. Points to Remember . . . . .	267
5.11. Exercise Problems . . . . .	269

**Chapter 6**

Design of State Feedback Controllers and Observers . . . . .	271
6.1. Introduction . . . . .	271
6.2. Controllability of Systems . . . . .	271
6.2.1. Criterion for Controllability for Continuous Systems . . . . .	272
6.2.2. Controllability of Canonical Forms . . . . .	277
6.2.3. Output Controllability . . . . .	280
6.2.4. Transformation to Controllable Canonical Form . . . . .	282
6.2.5. Controllability of Discrete Time Systems . . . . .	283
6.3. Observability of Systems . . . . .	285
6.3.1. Concept of Observability . . . . .	285
6.3.2. Criterion for Observability of a System . . . . .	285
6.3.3. Observability Using Canonical Forms . . . . .	288
6.3.4. Transformation to Observable Canonical Form . . . . .	291
6.3.5. Observability of Discrete Time Systems . . . . .	293
6.4. Significance of Controllability and Observability . . . . .	295
6.5. Transfer Function and Controllability/Observability . . . . .	295
6.6. State Feedback Controllers . . . . .	296
6.6.1. Pole Placement for Plants Represented in Phase Variable Form . . . . .	297
6.6.2. Determination of Feedback Gain $\mathbf{K}$ Using Ackerman's Formula . . . . .	301
6.7. State Feedback Controller Design with Any State Model . . . . .	305
6.8. Desired Location of Poles by Performance . . . . .	309
6.9. Design of Observers . . . . .	310
6.9.1. State Observers . . . . .	310
6.9.2. Separation Principle . . . . .	314
6.9.3. Reduced Order Observers . . . . .	315



6.10. Discrete Time Observers . . . . .	319
6.11. Additional Worked Examples . . . . .	320
6.12. Points to Remember . . . . .	335
6.13. Exercise Problems . . . . .	337

**Chapter 7**

Nonlinear Systems and Phase Plane Analysis . . . . .	341
7.1. Introduction . . . . .	341
7.2. Characteristics of Nonlinear Systems . . . . .	341
7.2.1. Superposition Principle is Not Valid . . . . .	341
7.2.2. Multiple Equilibrium States and Equilibrium Zones . . . . .	342
7.2.3. Limit Cycles or Sustained Oscillations . . . . .	342
7.2.4. Harmonics and Sub-Harmonic Oscillation in Output Under Sinusoidal Input . . . . .	342
7.2.5. Jump Phenomenon . . . . .	343
7.2.6. Frequency Entrainment or Synchronisation . . . . .	343
7.3. Methods of Analysis . . . . .	343
7.3.1. Linearisation Techniques . . . . .	344
7.3.2. Phase Plane Analysis . . . . .	344
7.3.3. Describing Function Analysis . . . . .	344
7.3.4. Lyapunov’s Method for Stability . . . . .	344
7.4. Classification of Nonlinearities . . . . .	345
7.4.1. Inherent or Intentional Nonlinearities . . . . .	345
7.4.2. Static and Dynamic Nonlinearity . . . . .	345
7.4.3. Functional and Piecewise Linear . . . . .	345
7.4.4. Memory Type (Multi-Valued) or Memory Less or Single-Valued . . . . .	345
7.5. Common Physical Nonlinearities . . . . .	346
7.5.1. Saturation or Limiter . . . . .	346
7.5.2. Dead Zone or Threshold . . . . .	347
7.5.3. Different Types of Relays . . . . .	347
7.5.4. Different Types of Springs . . . . .	348

**xiv Contents**

7.5.5. Different Types of Friction . . . . .	349
7.5.6. Back Lash in Gears . . . . .	350
7.6. Linearisation of Nonlinear Systems . . . . .	352
7.7. Phase Plane Analysis . . . . .	353
7.7.1. Phase Plane Method . . . . .	355
7.7.2. Phase Portraits . . . . .	357
7.7.3. Analytical Methods for the Construction of Phase Trajectories . . .	358
7.7.4. Graphical Method of Construction of Phase Trajectory—Isoclines Method . . . . .	361
7.7.5. Delta Method of Construction of Phase Trajectory . . . . .	366
7.7.6. Pell’s Method of Construction of Phase Trajectory . . . . .	369
7.8. Evaluation of Time on Phase Trajectory . . . . .	372
7.9. Analysis and Classification of Singular Points . . . . .	374
7.10. Limit Cycles on Phase Plane . . . . .	383
7.11. Extension to Systems with Piecewise Constant Inputs . . . . .	385
7.12. Additional Worked Examples . . . . .	387
7.13. Points to Remember . . . . .	403
7.14. Exercise Problems . . . . .	404

**Chapter 8**

Describing Function Analysis of Nonlinear Systems . . . . .	407
8.1. Introduction . . . . .	407
8.2. Basic Definition of Describing Function . . . . .	407
8.3. Basis of Describing Function Analysis . . . . .	409
8.4. Describing Function for Typical Nonlinearities . . . . .	410
8.4.1. Describing Function for Ideal Relay . . . . .	411
8.4.2. Relay with Dead Zone . . . . .	413
8.4.3. Simple Dead Zone . . . . .	414
8.4.4. Saturation or Limiter . . . . .	416
8.4.5. Relay with Hysteresis and Dead Zone . . . . .	417
8.4.6. Friction Controlled Backlash . . . . .	421

8.5. Application of Describing Function . . . . .	422
8.5.1. Closed Loop Stability Using Describing Function . . . . .	422
8.5.2. Stability of the Limit Cycles . . . . .	425
8.5.3. Accuracy of Describing Function Analysis . . . . .	426
8.5.4. Relative Stability from Describing Function . . . . .	434
8.5.5. Closed Loop Frequency Response . . . . .	434
8.6. Additional Worked Examples . . . . .	436
8.7. Points to Remember . . . . .	444
8.8. Exercise Problems . . . . .	446

**Chapter 9**

Stability of Systems . . . . .	449
9.1. Concept of Stability . . . . .	449
9.2. Equilibrium Points . . . . .	450
9.3. Stability in the Small and Stability in the Large . . . . .	451
9.4. Lyapunov’s Stability Definitions . . . . .	451
9.5. Local Linearisation and Stability in the Small . . . . .	454
9.6. Stability by the Method of Lyapunov . . . . .	457
9.6.1. First Method of Lyapunov . . . . .	457
9.6.2. Concept of Lyapunov’s Stability Theorems . . . . .	458
9.6.3. Sign Definiteness of Scalar Functions . . . . .	461
9.6.4. Lyapunov’s Stability Theorems . . . . .	464
9.7. Lyapunovs Method for Linear Time Invariant Systems . . . . .	467
9.7.1. Stability of Linear Continuous Time Systems . . . . .	468
9.7.2. Stability of Linear Discrete Time Systems . . . . .	470
9.8. Stability of Nonlinear Systems by Method of Lyapunov . . . . .	471
9.9. Krasovskii’s Theorem on Lyapunov Function . . . . .	472
9.9.1. Variable Gradient Method of Generating a Lyapunov Function . . . . .	473
9.10. Application of Lyapunov Function to Estimate Transients . . . . .	478
9.11. Additional Worked Examples . . . . .	480

9.12. Points to Remember . . . . . 485  
9.13. Exercise Problems . . . . . 487

**Chapter 10**

Introduction to Optimal Control . . . . . 489  
10.1. Introduction . . . . . 489  
10.2. Formulation of the Optimal Control Problem . . . . . 489  
    10.2.1. Characteristics of the Plant . . . . . 490  
    10.2.2. Requirements on the Plant . . . . . 490  
10.3. Typical Optimal Control Performance Measures . . . . . 491  
    10.3.1. Minimum Time Control Problem . . . . . 491  
    10.3.2. Minimum Energy Problem . . . . . 491  
    10.3.3. Minimum Fuel Problem . . . . . 492  
    10.3.4. State Regulator Problem . . . . . 492  
    10.3.5. Output Regulator Problem . . . . . 493  
    10.3.6. Tracking Problem . . . . . 493  
    10.3.7. Choice of Performance Measure . . . . . 493  
    10.3.8. Nature of Information about the Plant Supplied to the Controller . . . . . 493  
10.4. Optimal Control Based on Quadratic Performance Measures . . . . . 494  
    10.4.1. General Remarks on the Solution of Regulator Problem . . . . . 499  
10.5. Optimal Control System Design Using Second Method of Lyapunov . . . . . 499  
    10.5.1. Parameter Optimisation via Second Method of Lyapunov . . . . . 499  
10.6. Additional Worked Examples . . . . . 502  
10.7. Points to Remember . . . . . 508  
10.8. Exercise Problems . . . . . 509  
  
**Multiple Choice Questions** . . . . . 513  
**Answers to Exercise Problems** . . . . . 533  
**Appendix** . . . . . 543  
**References** . . . . . 553  
**Index** . . . . . 557

# Modern Control Engineering

*Dr. K. P. Mohandas*

Department of Electrical Engineering,  
National Institute of Technology,  
NIT Campus PO, Calicut



SANGUINE

Sanguine Technical Publishers  
Bangalore 560 016, India  
2006

# Contents

## Chapter 1

Review of Classical Control Theory . . . . .	1
1.1. Introduction . . . . .	1
1.2. Transfer Function Model . . . . .	1
1.3. Time Response Analysis . . . . .	3
1.3.1. Test Inputs for Time Response Analysis . . . . .	3
1.4. Performance Specifications in Time Domain . . . . .	7
1.5. Time Response of First Order Systems . . . . .	9
1.6. Time Response of Second Order Systems . . . . .	11
1.7. Response of Higher Order Systems . . . . .	14
1.8. Frequency Domain Analysis . . . . .	15
1.8.1. Root Locus Technique . . . . .	15
1.8.2. Frequency Response . . . . .	16
1.8.3. Frequency Domain Performance Measures . . . . .	22
1.8.4. Determination of Frequency Domain Measures . . . . .	25
1.9. Correlation Between Time Domain and Frequency Domain Specifications	27
1.9.1. Peak Resonance, Resonant Frequency and Bandwidth in terms of $\zeta$ and $\omega_n$ . . . . .	27
1.9.2. Bandwidth . . . . .	28
1.10. Stability of Linear Systems . . . . .	29
1.10.1. Bounded Input Bounded Output Stability . . . . .	29

**viii Contents**

1.10.2. Characteristic Equation Roots and Impulse Response . . . . . 29  
1.10.3. Stability from Polar Plot and Bode Plot . . . . . 29  
1.10.4. Determination of Stability—Routh Hurwitz Criterion . . . . . 32

**Chapter 2**

Conventional Controllers and Classical Design . . . . . 35  
2.1. Introduction . . . . . 35  
2.2. Types of Controllers . . . . . 35  
2.3. Compensating Networks . . . . . 38  
    2.3.1. Phase Lag Compensator . . . . . 43  
    2.3.2. Lead Lag Compensator . . . . . 44  
2.4. Design of Lead Compensator Using Bode Plot . . . . . 46  
2.5. Design of Lead Compensator Using Root Locus . . . . . 53  
2.6. Design of Lag Compensator Using Bode Plot . . . . . 58  
    2.6.1. Lag Compensator Design Using Bode Plot . . . . . 58  
2.7. Design of Lag Compensator Using Root Locus . . . . . 64  
2.8. System Design Using PI Controllers . . . . . 68  
2.9. Design of Lead-Lag Compensators . . . . . 71  
    2.9.1. Lead-Lag Design by Bode Plot Method . . . . . 71  
    2.9.2. Lead-Lag Compensation Using Root Locus . . . . . 74  
2.10. PID Controller Design and Tuning . . . . . 77  
    2.10.1. Ziegler-Nichols Rules for Tuning PID Controllers . . . . . 77  
2.11. Feedback Compensation . . . . . 81  
2.12. Comparison of Different Types of Controllers . . . . . 84  
2.13. Additional Worked Examples . . . . . 86  
2.14. Points to Remember . . . . . 99  
2.15. Exercise Problems . . . . . 101

**Chapter 3**

Discrete Data Control Systems . . . . . 103  
3.1. Introduction . . . . . 103

3.2. Basic Elements of a Discrete Data System . . . . .	103
3.3. Advantages of Discrete Data Systems . . . . .	105
3.4. Different Types of Discrete Time Systems . . . . .	106
3.4.1. Pulse Amplitude Modulated Systems . . . . .	106
3.4.2. Pulse Width Modulated Systems . . . . .	106
3.4.3. Pulse Frequency Modulated (PFM) Systems . . . . .	107
3.5. Examples of Discrete Data Systems . . . . .	109
3.5.1. Stepper Motor Control of Hard Disk Drives . . . . .	109
3.5.2. A Simplified Single-Axis Autopilot Control System . . . . .	109
3.5.3. A Digital Computer Controlled Rolling Mill Regulating System . . . . .	111
3.5.4. Computer Control of Turbine and Generator . . . . .	111
3.6. Sampling Process and Analysis . . . . .	112
3.6.1. Analysis of the Output of an Ideal Sampler in Time Domain . . . . .	112
3.6.2. Analysis of the Output of a Practical Sampler in Frequency Domain . . . . .	116
3.7. Sampling Theorem and Significance . . . . .	119
3.8. Data Reconstruction and Hold Circuits . . . . .	122
3.8.1. Zero Order Hold and Its Frequency Response . . . . .	123
3.8.2. First Order Hold and Data Reconstruction . . . . .	125
3.9. Mathematical Analysis of Discrete Data Systems . . . . .	128
3.9.1. $z$ -Transforms—Basic Definition . . . . .	128
3.9.2. $z$ -Transform Directly from Laplace Transform . . . . .	130
3.9.3. Example for Evaluation of $z$ -Transforms . . . . .	131
3.9.4. Properties of $z$ -Transforms . . . . .	133
3.9.5. Inverse $z$ -Transforms . . . . .	136
3.10. Solution of Difference Equation . . . . .	142
3.11. Pulse Transfer Function . . . . .	145
3.12. Block Diagram Reduction for Pulse Transfer Functions . . . . .	146
3.12.1. Cascaded Elements with Sampler in Between . . . . .	146
3.12.2. Cascaded Elements without Sampler in Between . . . . .	147
3.12.3. Closed Loop Control Systems . . . . .	148



**x Contents**

3.12.4. Zero-Order Hold and $G(s)$ in Cascade . . . . .	152
3.13. Time Response Analysis of Discrete Time Systems . . . . .	154
3.13.1. Time Response of Open Loop Systems . . . . .	154
3.13.2. Response of Closed Loop Systems . . . . .	155
3.14. Mapping Properties of Z-Transform . . . . .	157
3.15. Stability of Discrete Time Systems . . . . .	160
3.15.1. Routh-Hurwitz Criterion . . . . .	160
3.15.2. Jury's Test . . . . .	163
3.16. Response of Discrete Data Systems Between Sampling Instants . . . . .	165
3.17. Additional Worked Examples . . . . .	168
3.18. Points to Remember . . . . .	179
3.19. Exercise Problems . . . . .	182

**Chapter 4**

State Space Analysis of Systems . . . . .	185
4.1. Limitations of Classical Control . . . . .	185
4.2. Concept of State, State Space and State Variables . . . . .	186
4.3. State Model for Typical Systems . . . . .	188
4.3.1. Linear Systems . . . . .	188
4.3.2. Nonlinear Systems . . . . .	192
4.4. State Model of Linear Systems from Differential Equations . . . . .	193
4.5. State Variable Diagram and Block Diagram Representation of State Models	196
4.6. State Space Model for Physical Systems . . . . .	198
4.6.1. Electrical Circuits . . . . .	198
4.6.2. Mechanical Systems . . . . .	201
4.6.3. Electro-Mechanical Systems—DC Motors . . . . .	202
4.7. State Space Model from Transfer Functions . . . . .	205
4.7.1. Transfer Functions without Numerator Dynamics . . . . .	205
4.7.2. Transfer Function with Numerator Dynamics . . . . .	207
4.7.3. Series Decomposition . . . . .	211
4.7.4. Parallel Decomposition . . . . .	212

4.8. State Model for a Multi-Input Multi-Output System from Block Diagrams 216

4.9. Non-Uniqueness of State Space Model . . . . . 217

4.10. Transfer Function from State Model . . . . . 219

4.11. Canonical Models . . . . . 220

    4.11.1. Phase Variable Form or Controllable Canonical Model . . . . . 220

    4.11.2. Observable Canonical Model . . . . . 221

    4.11.3. Diagonal Canonical Model . . . . . 221

    4.11.4. Jordan Canonical Model . . . . . 222

4.12. Diagonalisation of a Matrix . . . . . 222

4.13. State Variable Description of Discrete Time Systems . . . . . 224

4.14. Additional Worked Examples . . . . . 228

4.15. Points to Remember . . . . . 238

4.16. Exercise Problems . . . . . 240

**Chapter 5**

Time Domain Analysis in State Space . . . . . 245

5.1. Solution of Time Invariant State Equation . . . . . 245

    5.1.1. Scalar Differential Equation—Homogeneous Case . . . . . 245

    5.1.2. Vector Matrix Differential Equation—Homogeneous Case . . . . . 246

    5.1.3. Laplace Transforms Method . . . . . 247

5.2. State Transition Matrix . . . . . 247

5.3. Solution of State Equation with Input . . . . . 249

5.4. Laplace Transform Approach . . . . . 251

5.5. State Transition Matrix from Cayleigh–Hamilton Theorem . . . . . 252

    5.5.1. Cayleigh–Hamilton Theorem . . . . . 253

    5.5.2. Computation of  $e^{At}$ —Method 1 . . . . . 253

    5.5.3. Computation of  $e^{At}$ —Method 2 . . . . . 254

5.6. Solution of Linear Time Varying State Equation . . . . . 258

5.7. Solution of State Equation in Canonical Forms . . . . . 260

    5.7.1. Diagonal Canonical Form and State Transition Matrix . . . . . 260

    5.7.2. Jordan Canonical Form . . . . . 261

**xii Contents**

5.8. Solution of Time Invariant Discrete Time State Equation . . . . .	262
5.9. Additional Worked Examples . . . . .	263
5.10. Points to Remember . . . . .	267
5.11. Exercise Problems . . . . .	269

**Chapter 6**

Design of State Feedback Controllers and Observers . . . . .	271
6.1. Introduction . . . . .	271
6.2. Controllability of Systems . . . . .	271
6.2.1. Criterion for Controllability for Continuous Systems . . . . .	272
6.2.2. Controllability of Canonical Forms . . . . .	277
6.2.3. Output Controllability . . . . .	280
6.2.4. Transformation to Controllable Canonical Form . . . . .	282
6.2.5. Controllability of Discrete Time Systems . . . . .	283
6.3. Observability of Systems . . . . .	285
6.3.1. Concept of Observability . . . . .	285
6.3.2. Criterion for Observability of a System . . . . .	285
6.3.3. Observability Using Canonical Forms . . . . .	288
6.3.4. Transformation to Observable Canonical Form . . . . .	291
6.3.5. Observability of Discrete Time Systems . . . . .	293
6.4. Significance of Controllability and Observability . . . . .	295
6.5. Transfer Function and Controllability/Observability . . . . .	295
6.6. State Feedback Controllers . . . . .	296
6.6.1. Pole Placement for Plants Represented in Phase Variable Form . . . . .	297
6.6.2. Determination of Feedback Gain $\mathbf{K}$ Using Ackerman's Formula . . . . .	301
6.7. State Feedback Controller Design with Any State Model . . . . .	305
6.8. Desired Location of Poles by Performance . . . . .	309
6.9. Design of Observers . . . . .	310
6.9.1. State Observers . . . . .	310
6.9.2. Separation Principle . . . . .	314
6.9.3. Reduced Order Observers . . . . .	315

6.10. Discrete Time Observers . . . . . 319  
 6.11. Additional Worked Examples . . . . . 320  
 6.12. Points to Remember . . . . . 335  
 6.13. Exercise Problems . . . . . 337

**Chapter 7**

Nonlinear Systems and Phase Plane Analysis . . . . . 341  
 7.1. Introduction . . . . . 341  
 7.2. Characteristics of Nonlinear Systems . . . . . 341  
     7.2.1. Superposition Principle is Not Valid . . . . . 341  
     7.2.2. Multiple Equilibrium States and Equilibrium Zones . . . . . 342  
     7.2.3. Limit Cycles or Sustained Oscillations . . . . . 342  
     7.2.4. Harmonics and Sub-Harmonic Oscillation in Output  
         Under Sinusoidal Input . . . . . 342  
     7.2.5. Jump Phenomenon . . . . . 343  
     7.2.6. Frequency Entrainment or Synchronisation . . . . . 343  
 7.3. Methods of Analysis . . . . . 343  
     7.3.1. Linearisation Techniques . . . . . 344  
     7.3.2. Phase Plane Analysis . . . . . 344  
     7.3.3. Describing Function Analysis . . . . . 344  
     7.3.4. Lyapunov’s Method for Stability . . . . . 344  
 7.4. Classification of Nonlinearities . . . . . 345  
     7.4.1. Inherent or Intentional Nonlinearities . . . . . 345  
     7.4.2. Static and Dynamic Nonlinearity . . . . . 345  
     7.4.3. Functional and Piecewise Linear . . . . . 345  
     7.4.4. Memory Type (Multi-Valued) or Memory Less or Single-Valued . . 345  
 7.5. Common Physical Nonlinearities . . . . . 346  
     7.5.1. Saturation or Limiter . . . . . 346  
     7.5.2. Dead Zone or Threshold . . . . . 347  
     7.5.3. Different Types of Relays . . . . . 347  
     7.5.4. Different Types of Springs . . . . . 348

**xiv Contents**

7.5.5. Different Types of Friction . . . . .	349
7.5.6. Back Lash in Gears . . . . .	350
7.6. Linearisation of Nonlinear Systems . . . . .	352
7.7. Phase Plane Analysis . . . . .	353
7.7.1. Phase Plane Method . . . . .	355
7.7.2. Phase Portraits . . . . .	357
7.7.3. Analytical Methods for the Construction of Phase Trajectories . . .	358
7.7.4. Graphical Method of Construction of Phase Trajectory—Isoclines Method . . . . .	361
7.7.5. Delta Method of Construction of Phase Trajectory . . . . .	366
7.7.6. Pell’s Method of Construction of Phase Trajectory . . . . .	369
7.8. Evaluation of Time on Phase Trajectory . . . . .	372
7.9. Analysis and Classification of Singular Points . . . . .	374
7.10. Limit Cycles on Phase Plane . . . . .	383
7.11. Extension to Systems with Piecewise Constant Inputs . . . . .	385
7.12. Additional Worked Examples . . . . .	387
7.13. Points to Remember . . . . .	403
7.14. Exercise Problems . . . . .	404

**Chapter 8**

Describing Function Analysis of Nonlinear Systems . . . . .	407
8.1. Introduction . . . . .	407
8.2. Basic Definition of Describing Function . . . . .	407
8.3. Basis of Describing Function Analysis . . . . .	409
8.4. Describing Function for Typical Nonlinearities . . . . .	410
8.4.1. Describing Function for Ideal Relay . . . . .	411
8.4.2. Relay with Dead Zone . . . . .	413
8.4.3. Simple Dead Zone . . . . .	414
8.4.4. Saturation or Limiter . . . . .	416
8.4.5. Relay with Hysteresis and Dead Zone . . . . .	417
8.4.6. Friction Controlled Backlash . . . . .	421

8.5.	Application of Describing Function . . . . .	422
8.5.1.	Closed Loop Stability Using Describing Function . . . . .	422
8.5.2.	Stability of the Limit Cycles . . . . .	425
8.5.3.	Accuracy of Describing Function Analysis . . . . .	426
8.5.4.	Relative Stability from Describing Function . . . . .	434
8.5.5.	Closed Loop Frequency Response . . . . .	434
8.6.	Additional Worked Examples . . . . .	436
8.7.	Points to Remember . . . . .	444
8.8.	Exercise Problems . . . . .	446

**Chapter 9**

	Stability of Systems . . . . .	449
9.1.	Concept of Stability . . . . .	449
9.2.	Equilibrium Points . . . . .	450
9.3.	Stability in the Small and Stability in the Large . . . . .	451
9.4.	Lyapunov’s Stability Definitions . . . . .	451
9.5.	Local Linearisation and Stability in the Small . . . . .	454
9.6.	Stability by the Method of Lyapunov . . . . .	457
9.6.1.	First Method of Lyapunov . . . . .	457
9.6.2.	Concept of Lyapunov’s Stability Theorems . . . . .	458
9.6.3.	Sign Definiteness of Scalar Functions . . . . .	461
9.6.4.	Lyapunov’s Stability Theorems . . . . .	464
9.7.	Lyapunovs Method for Linear Time Invariant Systems . . . . .	467
9.7.1.	Stability of Linear Continuous Time Systems . . . . .	468
9.7.2.	Stability of Linear Discrete Time Systems . . . . .	470
9.8.	Stability of Nonlinear Systems by Method of Lyapunov . . . . .	471
9.9.	Krasovskii’s Theorem on Lyapunov Function . . . . .	472
9.9.1.	Variable Gradient Method of Generating a Lyapunov Function . . . . .	473
9.10.	Application of Lyapunov Function to Estimate Transients . . . . .	478
9.11.	Additional Worked Examples . . . . .	480

9.12. Points to Remember . . . . . 485  
9.13. Exercise Problems . . . . . 487

**Chapter 10**

Introduction to Optimal Control . . . . . 489  
10.1. Introduction . . . . . 489  
10.2. Formulation of the Optimal Control Problem . . . . . 489  
    10.2.1. Characteristics of the Plant . . . . . 490  
    10.2.2. Requirements on the Plant . . . . . 490  
10.3. Typical Optimal Control Performance Measures . . . . . 491  
    10.3.1. Minimum Time Control Problem . . . . . 491  
    10.3.2. Minimum Energy Problem . . . . . 491  
    10.3.3. Minimum Fuel Problem . . . . . 492  
    10.3.4. State Regulator Problem . . . . . 492  
    10.3.5. Output Regulator Problem . . . . . 493  
    10.3.6. Tracking Problem . . . . . 493  
    10.3.7. Choice of Performance Measure . . . . . 493  
    10.3.8. Nature of Information about the Plant Supplied to the Controller . . . . . 493  
10.4. Optimal Control Based on Quadratic Performance Measures . . . . . 494  
    10.4.1. General Remarks on the Solution of Regulator Problem . . . . . 499  
10.5. Optimal Control System Design Using Second Method of Lyapunov . . . . . 499  
    10.5.1. Parameter Optimisation via Second Method of Lyapunov . . . . . 499  
10.6. Additional Worked Examples . . . . . 502  
10.7. Points to Remember . . . . . 508  
10.8. Exercise Problems . . . . . 509  
  
**Multiple Choice Questions** . . . . . 513  
**Answers to Exercise Problems** . . . . . 533  
**Appendix** . . . . . 543  
**References** . . . . . 553  
**Index** . . . . . 557