NONLINEAR MODEL: CELL FORMATION

0-1 Integer Programming Formulation

System Requirements and Symbols

\( \alpha_i \) - Number of intercell transfers due to part \( i \)

\( \beta_i \) - Demand of part \( i \)

\( \gamma_i \) - Unit intercell transfer cost of part \( i \)

\( M_{ik} \) - Machine type required for the operation \( k \) of the part \( i \)

\( i \) – index of parts, \( i = 1, 2, ..., N_p \)

\( j \) – Index of cells, \( j = 1, 2, ..., N_c \)

\( k \) - Index of operation, \( k = 1, 2, ..., N_i \)

\( M \) – index of machines, \( M = 1, 2, ..., N_m \)

\( N_i \) - Number of operations required to complete the processing requirement of part \( i \)

\( m_{\text{min}} \) - Minimum number of machine types per cell

\( m_{\text{max}} \) - Maximum number of machine types per cell

\( \theta \) - minimum number of parts per family

\( Z \) – Intercell material handling cost

\( x_{ij} = \begin{cases} 1 & \text{if part } i \text{ is assigned to cell } j \\ 0 & \text{Otherwise} \end{cases} \)

\( \sigma_{M_{ik},j} = \begin{cases} 1 & \text{if machine type } M_{ik} \text{ is assigned to cell } j \\ 0 & \text{Otherwise} \end{cases} \)

- Number of part families = Number of machine cells.
- A machine type is assigned to only one cell.
- When a part is assigned to a cell and some of the machines required for completing the operation required for this part is available in another cell, then the intercell transfer calculation involves the following:
  - Consecutive operations of a part in a non-assigned cell create intracell moves. Each such intracell move is considered as an intercell move.
This may be interpreted as that any movement of a part other than in the assigned cell has to be penalized. (That is, if a part is assigned to a cell then all its operation should be completed in the cell in which it is assigned.)

The intercell calculation equation below takes into account this fact

Formulation

\[ Z = \sum_{i=1}^{N_p} \alpha_i \beta_i \gamma_i \quad \text{[1]} \]

Minimize

\[ \alpha_i = \sum_{j=1}^{N_c} \sum_{k=1}^{N_i-1} x_{ij} \left[ 1 - \sigma M_{ik}, j \sigma M_{i(k+1)}, j \right] \quad \forall i \quad \text{[2]} \]

Subjected to

\[ \sum_{j=1}^{N_c} x_{ij} = 1 \quad \forall i \quad \text{[3]} \]

\[ \sum_{i=1}^{N_p} x_{ij} \geq \theta \quad \forall j \quad \text{[4]} \]

\[ \sum_{k=1}^{N_i} \sum_{j=1}^{N_c} \sigma M_{ik}, j = N_i \quad \forall i \quad \text{[5]} \]

\[ \sum_{M=1}^{N_m} \sigma M_j \geq m_{\text{min}} \quad \forall j \quad \text{[6]} \]

\[ \sum_{M=1}^{N_m} \sigma M_j \leq m_{\text{max}} \quad \forall j \quad \text{[7]} \]

\[ \sigma M_{ik}, j, x_{ij} = \{0,1\} \quad \forall i, k, j \]
Example Problem

Minimize the intercell movement for the 5 machine 4 part problem given below.

\( m_{\text{min}} = 1 \) (Minimum number of machine types per cell)

\( m_{\text{max}} = 3 \) (Maximum number of machine types per cell)

\( \theta = 1 \) (Minimum number of parts per family)

\( N_c = 2 \) (Number of cells)

\( \gamma = 1 \) (Unit intercell transfer cost)

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<thead>
<tr>
<th>Machines</th>
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<tbody>
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<tr>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>( \beta_i )</th>
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<td>2</td>
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Table: Operations required for parts

The above table provides the machine sequence required to complete the various operations of parts. For example, first operation of part 1 is in machine 3, second operation is in machine 2, third operation is in machine 5 and so on. Hence, the machine sequence for the part 1 is 3-2-5-4.

Formulation tips

Number of terms in the objective function = \( \sum_{i=1}^{N_p} N_c (N_i - 1) \)

Number of constraints = \( N_p + N_c + N_p + N_c + N_c \)

Number of decision variables = \( N_p \times N_c + N_m \times N_c \)
Formulation for the above problem

Objective function

\[ Z = (x_{11}(1 - \sigma_{31} \sigma_{21}) + x_{12}(1 - \sigma_{21} \sigma_{51}) + x_{13}(1 - \sigma_{51} \sigma_{41})) 
+ (x_{12}(1 - \sigma_{32} \sigma_{22}) + x_{22}(1 - \sigma_{22} \sigma_{52}) + x_{12}(1 - \sigma_{52} \sigma_{42})) \leq 10 \]
\[ + (x_{21}(1 - \sigma_{41} \sigma_{12}) + x_{21}(1 - \sigma_{11} \sigma_{31}) + x_{21}(1 - \sigma_{31} \sigma_{21})) 
+ (x_{22}(1 - \sigma_{42} \sigma_{12}) + x_{22}(1 - \sigma_{12} \sigma_{32}) + x_{22}(1 - \sigma_{32} \sigma_{22})) \leq 15 \]
\[ + (x_{31}(1 - \sigma_{31} \sigma_{21}) + x_{31}(1 - \sigma_{21} \sigma_{11}) 
+ x_{32}(1 - \sigma_{32} \sigma_{22}) + x_{32}(1 - \sigma_{22} \sigma_{12})) \leq 20 \]
\[ + (x_{41}(1 - \sigma_{21} \sigma_{41}) + x_{41}(1 - \sigma_{41} \sigma_{51}) + x_{41}(1 - \sigma_{51} \sigma_{11}) 
+ x_{42}(1 - \sigma_{22} \sigma_{42}) + x_{42}(1 - \sigma_{42} \sigma_{52}) + x_{42}(1 - \sigma_{52} \sigma_{12})) \leq 10 \]

Subjected to

Constraints due to eqn. 3
\[ x_{11} + x_{12} = 1 \]
\[ x_{21} + x_{22} = 1 \]
\[ x_{31} + x_{32} = 1 \]
\[ x_{41} + x_{42} = 1 \]

Constraints due to eqn. 4
\[ x_{11} + x_{21} + x_{31} + x_{41} \geq 1 \]
\[ x_{12} + x_{22} + x_{32} + x_{42} \geq 1 \]

Constraints due to eqn. 5
\[ \sigma_{31} + \sigma_{32} + \sigma_{21} + \sigma_{22} + \sigma_{51} + \sigma_{52} + \sigma_{41} + \sigma_{42} = 4 \]
\[ \sigma_{41} + \sigma_{42} + \sigma_{11} + \sigma_{12} + \sigma_{31} + \sigma_{32} + \sigma_{21} + \sigma_{22} = 4 \]
\[ \sigma_{31} + \sigma_{32} + \sigma_{21} + \sigma_{22} + \sigma_{11} + \sigma_{12} = 3 \]
\[ \sigma_{21} + \sigma_{22} + \sigma_{41} + \sigma_{42} + \sigma_{51} + \sigma_{52} + \sigma_{11} + \sigma_{12} = 4 \]

Constraints due to eqn. 6
\[ \sigma_{11} + \sigma_{21} + \sigma_{31} + \sigma_{41} + \sigma_{51} \geq 1 \]
\[ \sigma_{12} + \sigma_{22} + \sigma_{32} + \sigma_{42} + \sigma_{52} \geq 1 \]

Constraints due to eqn. 7
\[ \sigma_{11} + \sigma_{21} + \sigma_{31} + \sigma_{41} + \sigma_{51} \leq 3 \]
\[ \sigma_{12} + \sigma_{22} + \sigma_{32} + \sigma_{42} + \sigma_{52} \leq 3 \]

Integrality constraints
\[ x_{ij}, \sigma_{Mj} = \{0,1\} \]
A feasible solution to this problem is

$$\sigma_{11} = 1, \sigma_{21} = 1, \sigma_{31} = 1, \sigma_{42} = 1, \sigma_{52} = 1$$

$$x_{12} = 1, x_{21} = 1, x_{31} = 1, x_{42} = 1$$

All other decision variables have value equal to zero

This has an objective function value of 55

The corresponding cells and part families are as follows:

Cell 1 = \{1, 2, 3\} & Part-family 1 = \{2, 3\}

Cell 2 = \{4, 5\} & Part-family 2 = \{1, 4\}
QUESTIONS:

1. Intercell moves create long flow paths for part processing, why? Are intercell moves desirable? Substantiate your answer.

2. Write down the integer programming formulation of the cell formation problem. The manufacturing environment consists of a deterministic product mix for the planning horizon of the design. Minimum number of types of machines and maximum number of types of machines in a cell are constrained. The number of parts in a part family is also constrained. The route sheet and demand information of the part mix are available.

3. Define a decision variable of a mathematical programming problem which shows a part assignment to a cell (part family). A part can be assigned only to a part family. Generate all such constraints.

4. \[ \alpha_i = \sum_{j=1}^{N_c} \sum_{k=1}^{N_{ij}} x_{ij} \left[ 1 - \sigma_{M_{ik}, j} \sigma_{M_{(k+1), j}} \right] \quad \forall i \]

This equation is used for intercell movement calculation for a single period cellular manufacturing system design problem. The notations and parameters used are as follows:

- \( \alpha_i \) is the number of intercell movements due to part \( i \).
- \( N_c = 2 \) (Number of cells)
- \( N_{ij} \) is number of operations for part \( i \).
- \( \beta_i \) is the demand of part \( i \).
- \( \sigma_{M_{ik}, j} = \begin{cases} 1 & \text{if machine type } M_{ik} \text{ is assigned to cell } j \\ 0 & \text{Otherwise} \end{cases} \)
- \( x_{ij} = \begin{cases} 1 & \text{if part } i \text{ is assigned to cell } j \\ 0 & \text{Otherwise} \end{cases} \)

- \( i \) – index of parts, \( i = 1, 2, \ldots, N_p \)
- \( j \) – index of cells, \( j = 1, 2, \ldots, N_c \)
- \( k \) – index of operations

The operation sequence for 4 parts which are processed in 5 machines is provided in the table given below.
A feasible solution for the above problem is given below.

\[ \sigma_{11} = 1, \sigma_{21} = 1, \sigma_{31} = 1, \sigma_{42} = 1, \sigma_{52} = 1 \]

\[ x_{12} = 1, x_{21} = 1, x_{31} = 1, x_{42} = 1 \]

Determine the intercell movements using the above equation.

5. \[ \sum_{i=1}^{N_p} x_{ij} \geq \theta \ \forall j \ldots \ [1] \]
   This equation shows a constraint for a single period cellular manufacturing system design problem. The notations and parameters used are as follows:
   \[ \theta = 1 \] (Minimum number of parts per family),
   \[ N_c = 2 \] (Number of cells)
   \[ N_p = 4 \] (Number of parts)
   \[ x_{ij} = \begin{cases} 1 & \text{if part } i \text{ is assigned to cell } j \\ 0 & \text{Otherwise} \end{cases} \]
   \[ i \] – index of parts, \( i = 1, 2, \ldots, N_p \)
   \[ j \] – Index of cells, \( j = 1, 2, \ldots, N_c \)

Write down all the constraints generated by the equation [1] for the given data.

6. The matrix below shows the operation sequence for 8 products produced in a production system. For example, the operation sequence of part 1 is 1-2-3-4 and the corresponding machine sequence is C-D-H-F. That is, the machine type used for the first operation of part 1 is C. Similarly, machine types required for operations 2, 3 and 4 can be established.
A cell formation is considered for this production system with the following features:

\[ \theta = 3 \text{ (Minimum number of parts per family), } N_c = 2 \text{ (Number of cells)} \]

A mathematical model for cell formation of the above production system is developed which contains the following constraints.

\[ \sum_{i=1}^{N_p} x_{ij} \geq \theta \quad \forall j \quad \cdots [1] \]

\[ \sum_{k=1}^{N_i} \sum_{j=1}^{N_c} \sigma_{M_{ik},j} = N_i \quad \forall i \quad \cdots [2] \]

The notations used in the above constraints are as follows:

\[ x_{ij} = \begin{cases} 
1 & \text{if part } i \text{ is assigned to cell } j \\
0 & \text{Otherwise}
\end{cases} \]

\[ \sigma_{M_{ik},j} = \begin{cases} 
1 & \text{if machine type } M_{ik} \text{ is assigned to cell } j \\
0 & \text{Otherwise}
\end{cases} \]

\( N_p \) - Number of parts

\( N_i \) - Number of operations required to complete the processing requirement of part \( i \)

\( M_{ik} \) - Machine type required for the operation \( k \) of the part \( i \)

\( k \) - Index of operation, \( k = 1, 2, \ldots, N_i \)

\( M \) – index of machines, \( M = 1, 2, \ldots, N_m \)

\( (M = 1 \text{ is corresponding machine A, } 2 \text{ corresponds to B, and so on}) \)

\( i \) – index of parts, \( i = 1, 2, \ldots, N_p \)

\( j \) – Index of cells, \( j = 1, 2, \ldots, N_c \)

Write down all the constraints generated by [1] and [2] for the data given above.